Stabilisation of Highly Nonlinear Hybrid Stochastic Differential Delay Equations by Delay Feedback Control

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(Joint work with Xiaoyue Li)
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   - Notation and standing hypotheses
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Hybrid stochastic differential delay equations (SDDEs) whose coefficients depend on the states of continuous-time Markov chains (also known as SDDEs with Markovian switching) appear in many branches of science and industry.

One of the important issues in the study of hybrid SDDEs is the analysis of stability.

In particular, the stability of highly nonlinear hybrid SDDEs has recently become one of the most popular topics.

Consider an unstable hybrid SDDE

$$dx(t) = f(x(t), x(t-\delta), r(t), t)dt + g(x(t), x(t-\delta), r(t), t)dB(t), \tag{1.1}$$

where the state $x(t)$ takes values in R^n and the mode $r(t)$ is a Markov chain taking values in a finite space $S = \{1, 2, \cdots, N\}$, $B(t)$ is a Brownian motion, δ is a positive constant which stands for the time delay of the system, and f and g are referred to as the drift and diffusion coefficient, respectively.

In order to make this given unstable system become stable, it is classical to find a feedback control $u(x(t), r(t), t)$, based on the current state $x(t)$, for the controlled system

$$\begin{aligned} dx(t) &= [f(x(t), x(t-\delta), r(t), t) + u(x(t), r(t), t)]dt \\ &\quad + g(x(t), x(t-\delta), r(t), t)dB(t) \end{aligned} \tag{1.2}$$

to become stable.

However, there is always a time lag τ between the time when the observation of the state is made and the time when the feedback control reaches the system.

It takes 1.28 seconds for a radio signal from the moon to reach the earth. If the time unit is of year, then 1.28 seconds $= 4.0576 * 10^{-8}$ year.

The real controlled system should therefore be

$$\begin{aligned} dx(t) &= [f(x(t), x(t-\delta), r(t), t) + u(x(t-\tau), r(t), t)]dt \\ &\quad + g(x(t), x(t-\delta), r(t), t)dB(t) \end{aligned} \tag{1.3}$$

to be stable.

It is traditional to design τ to be extremely small, as almost everyone has believed:

The real controlled system (1.3) should behave in the same way as the theoretical controlled SDDE (1.2) does as long as τ is extremely small.

Question: *Can we really take this for granted?*

Answer: *No.*

- The scalar SDE

$$dx(t) = -x(t)dt + x^2(t)dB(t)$$

is not exponentially stable in mean square.

- The non-delay feedback controlled SDE

$$dx(t) = [-x(t) - 2x^3(t)]dt + x^2(t)dB(t)$$

is exponentially stable in mean square.

- However, the corresponding delay feedback controlled SDDE

$$dx(t) = [-x(t) - 2x^3(t-\tau)]dt + x^2(t)dB(t)$$

is NOT exponentially stable in mean square for any $\tau > 0$, no matter how small τ is.

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is NOT exponentially stable in mean square for any $\tau > 0$, no matter how small τ is.

The counterexample seems to indicate it is impossible to use delay feedback controls, even when the given system is an SDE.

Mao, Lam and Huang (2008) were the first to study this stabilisation problem by the delay feedback control. They showed

$$dx(t) = [A(r(t))x(t) + C(r(t))x(t)]dt + G(r(t))x(t)dB(t)$$

is exponentially stable in mean square if and only if there is a positive constant τ^* such that

$$dx(t) = [A(r(t))x(t) + C(r(t))x(t-\tau)]dt + G(r(t))x(t)dB(t)$$

is exponentially stable in mean square provided $\tau \le \tau^*$.

Hu, Liu, Deng and Mao (2020): Under the global Lipschitz continuity of f, g, u,

$$dx(t) = [f(x(t), r(t)) + u(x(t), r(t))]dt + g(x(t), r(t))dB(t)$$

is exponentially stable in pth moment ($p > 0$) if and only if there is a positive constant τ^* such that

$$dx(t) = [f(x(t), r(t)) + u(x(t - \tau), r(t))]dt + g(x(t), r(t))dB(t)$$

is exponentially stable in pth moment provided $\tau \le \tau^*$.

Lu, Hu and Mao (2018) made a significant progress by establishing a new theory on the stabilisation by delay feedback control for

$$dx(t) = [f(x(t), r(t)) + u(x(t-\tau), r(t))]dt + g(x(t), r(t))dB(t)$$

where f and g do NOT satisfy the linear growth condition, namely they are highly nonlinear.

The aim of this talk is to develop their theory further for highly nonlinear hybrid SDDEs.

We highlight a few significant features in comparison with Lu, Hu and Mao (2018):

- Under some standing hypotheses we will propose a number of rules to stabilise the given SDDE. We will explain how to design the delay feedback control to satisfy these rules and these discussions will also reveal that there are many such delay feedback controls available. Such developments are totally different from the study in Lu, Hu and Mao (2018).
- The stabilisation of SDDEs discussed in this paper is an infinite-dimensional problem while that of SDEs in Lu, Hu and Mao (2018) is finite-dimensional.
- The mathematical analysis of the infinite-dimensional problem in this paper is much harder than that of a finite-dimensional one in Lu, Hu and Mao (2018).

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Outline

If A is a vector or matrix, its transpose is denoted by A^T. For $x \in R^n$, $|x|$ denotes its Euclidean norm. If A is a matrix, we let $|A| = \sqrt{\mathrm{trace}(A^T A)}$ be its trace norm. If A is a symmetric real-valued matrix ($A = A^T$), denote by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ its smallest and largest eigenvalue, respectively. By $A \le 0$ and $A < 0$, we mean A is non-positive and negative definite, respectively. Let $R_+ = [0, \infty)$. For $h > 0$, denote by $C([-h, 0]; R^n)$ the family of continuous functions φ from $[-h, 0] \to R^n$ with the norm $\|\varphi\| = \sup_{-h \le u \le 0} |\varphi(u)|$. Denote by $C(R^n; R_+)$ the family of continuous functions from R^n to R_+. If both a, b are real numbers, then $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$. If A is a subset of Ω, denote by I_A its indicator function; that is, $I_A(\omega) = 1$ if $\omega \in A$ and 0 otherwise.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered complete probability space with a filtration satisfying the usual conditions (i.e., it is right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Let $B(t) = (B_1(t), \cdots, B_m(t))^T$ be an m-dimensional Brownian motion defined on the probability space. Let $r(t)$, $t \geq 0$, be a right-continuous Markov chain on the same probability space taking values in a finite state space $S = \{1, 2, \cdots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$\mathbb{P}\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while $\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}$. We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $B(\cdot)$ under $\mathbb{P}$.

Suppose that the underlying system is described by the nonlinear hybrid SDDE (1.1) with the initial data

$$\{x(t) : -\delta \le t \le 0\} = \xi \in C([-\delta, 0]; R^n), \tag{1.4}$$

where the coefficients $f : R^n \times R^n \times S \times R_+ \to R^n$ and $g : R^n \times R^n \times S \times R_+ \to R^{n \times m}$ are Borel measurable functions and locally Lipschitz continuous.

Assumption 1

Assume that for any real number $b > 0$, there exists a positive constant K_b such that

$$|f(x,y,i,t) - f(\bar{x},\bar{y},i,t)| \vee |g(x,y,i,t) - g(\bar{x},\bar{y},i,t)| \le K_b(|x-\bar{x}| + |y-\bar{y}|) \quad (1.5)$$

for all $x, y, \bar{x}, \bar{y} \in R^n$ with $|x| \vee |y| \vee |\bar{x}| \vee |\bar{y}| \le b$ and all $(i,t) \in S \times R_+$. Assume moreover that there exist constants $K > 0$, $q_1 > 1$ and $q_i \ge 1$ $(2 \le i \le 4)$ such that

$$\begin{aligned} |f(x,y,i,t)| &\le K(|x| + |y| + |x|^{q_1} + |y|^{q_2}), \\ |g(x,y,i,t)| &\le K(|x| + |y| + |x|^{q_3} + |y|^{q_4}) \end{aligned} \quad (1.6)$$

for all $(x,y,i,t) \in R^n \times R^n \times S \times R_+$.

For the convenience of the study in this paper we let $q_1 > 1$ but essentially we need only $\max_{1 \le i \le 4} q_i > 1$ as we are here interested in hybrid SDDEs without the linear growth condition. We will refer to condition (1.6) as the polynomial growth condition.

It is known that Assumption 1 only guarantees that the hybrid SDDE (1.1) has a unique maximal local solution, which may explode to infinity at a finite time (see, e.g., Mao and Yuan 2006). To avoid such a possible explosion, we need to impose another Khasminskii-type condition.

Assumption 2

Assume that there exist positive constants $p, q, \alpha_1, \alpha_2, \alpha_3$ *such that* $\alpha_2 > \alpha_3$ *and*

$$q > (p + q_1 - 1) \vee (2(q_1 \vee q_2 \vee q_3 \vee q_4)), \tag{1.7}$$

$$p \geq 2(q_1 \vee q_2 \vee q_3 \vee q_4) - q_1 + 1, \tag{1.8}$$

(where $q_1, \cdots, q_4$ *have been specified in Assumption 1) while for all* $(x, i, t) \in R^n \times S \times R_+$,

$$x^T f(x, y, i, t) + \frac{q-1}{2}|g(x, y, i, t)|^2 \leq \alpha_1(|x|^2 + |y|^2) - \alpha_2|x|^p + \alpha_3|y|^p. \tag{1.9}$$

Theorem 3

Under Assumptions 1 and 2, equation (1.1) with the initial data (1.4) has a unique global solution $x(t)$ *on* $[-\delta, \infty)$ *which satisfies*

$$\sup_{-\delta \le t < \infty} \mathbb{E}|x(t)|^q < \infty. \tag{1.10}$$

Outline

Although the solution of the hybrid SDDE (1.1) is bounded under Assumptions 1 and 2, the equation may not be stable.

In this case, we are required to design a delay feedback control $u(x(t-\tau), r(t), t)$ for the controlled equation (1.3) to become stable.

Here the control function $u : R^n \times S \times R_+ \to R^n$ is Borel measurable, while we shall assume $\tau \le \delta$ (it is possible to allow $\tau > \delta$ but the calculations will become more complicated).

We shall propose a number of rules for the control function u to meet for the stabilisation purpose.

Rule 4

There exists a positive number β *such that*

$$|u(x, i, t) - u(y, i, t)| \le \beta |x - y| \tag{2.1}$$

for all $x, y \in R^n$, $i \in S$ *and* $t \ge 0$. *Moreover, for the stability purpose, we require that* $u(0, i, t) \equiv 0$.

Theorem 5

Let Assumptions 1 and 2 hold. If the control function u satisfies Rule 4, then the controlled SDDE (1.3) with the initial data (1.4) has a unique global solution $x(t)$ *on* $[-\tau, \infty)$ *which satisfies*

$$\sup_{-\tau \le t < \infty} \mathbb{E}|x(t)|^q < \infty. \tag{2.2}$$

Rule 6

Design the control function $u : R^n \times S \times R_+ \to R^n$ *so that we can find real numbers* $a_i, \ \bar{a}_i$, *positive numbers* $c_i, \ \bar{c}_i$ *and nonnegative numbers* $b_i, \ \bar{b}_i, \ d_i, \ \bar{d}_i$ *(*$i \in S$*) such that*

$$x^T[f(x,y,i,t)+u(x,i,t)]+\frac{1}{2}|g(x,y,i,t)|^2 \le a_i|x|^2+b_i|y|^2-c_i|x|^p+d_i|y|^p, \tag{2.3}$$

$$x^T[f(x,y,i,t)+u(x,i,t)]+\frac{q_1}{2}|g(x,y,i,t)|^2 \le \bar{a}_i|x|^2+\bar{b}_i|y|^2-\bar{c}_i|x|^p+\bar{d}_i|y|^p \tag{2.4}$$

for all $(x,y,i,t) \in R^n \times R^n \times S \times R_+$*; while both*

$$\begin{aligned} \mathcal{A}_1 &:= -2\mathrm{diag}(a_1,\cdots,a_N)-\Gamma, \\ \textit{and} \ \ \mathcal{A}_2 &:= -(q_1+1)\mathrm{diag}(\bar{a}_1,\cdots,\bar{a}_N)-\Gamma \end{aligned} \tag{2.5}$$

Continuation of Rule 6

are nonsingular M-matrices; and moreover,

$$1 > \gamma_1, \ \gamma_2 > \gamma_3, \ 1 > \gamma_4, \ \gamma_5 > \gamma_6, \tag{2.6}$$

where

$$\begin{aligned} (\theta_1, \cdots, \theta_N)^T &= \mathcal{A}_1^{-1}(1, \cdots, 1)^T, \\ (\bar\theta_1, \cdots, \bar\theta_N)^T &= \mathcal{A}_2^{-1}(1, \cdots, 1)^T, \end{aligned} \tag{2.7}$$

$$\begin{aligned} \gamma_1 &= \max_{i\in S} 2\theta_i b_i, & \gamma_2 &= \min_{i\in S} 2\theta_i c_i, \\ \gamma_3 &= \max_{i\in S} 2\theta_i d_i, & \gamma_4 &= \max_{i\in S}(q_1+1)\bar\theta_i \bar b_i, \\ \gamma_5 &= \min_{i\in S}(q_1+1)\bar\theta_i \bar c_i, & \gamma_6 &= \max_{i\in S}(q_1+1)\bar\theta_i \bar d_i. \end{aligned} \tag{2.8}$$

Let us explain that there are lots of such control functions available under Assumption 2. For example, in the case when the state $x(t)$ of the given SDDE (1.1) is observable in any mode $i \in S$ (otherwise it is more complicated), we could, for example, design the control function $u(x, i, t) = Ax^T$, where A is a symmetric $n \times n$ real-valued negative-definite matrix such that $\lambda_{\max}(A) \le -(\kappa + 1)\alpha_1$ with $\kappa > 1$. Then

$$x^T u(x, i, t) \le -(\kappa + 1)\alpha_1 |x|^2, \quad \forall (x, i, t) \in R^n \times S \times R_+.$$

By Assumption 2, in particular, noting $q - 1 \ge q_1 > 1$, we further have

$$
\begin{aligned}
&x^T[f(x,y,i,t)+u(x,i,t)]+\frac{1}{2}|g(x,i,t)|^2\\
&\qquad\le -\kappa\alpha_1|x|^2+\alpha_1|y|^2-\alpha_2|x|^p+\alpha_3|y|^p,\\
&x^T[f(x,i,t)+u(x,i,t)]+\frac{q_1}{2}|g(x,i,t)|^2\\
&\qquad\le -\kappa\alpha_1|x|^2+\alpha_1|y|^2-\alpha_2|x|^p+\alpha_3|y|^p.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
\mathcal{A}_1 &= 2\kappa\,\mathrm{diag}(\alpha_1,\cdots,\alpha_1)-\Gamma,\\
\mathcal{A}_2 &= \kappa(q_1+1)\,\mathrm{diag}(\alpha_1,\cdots,\alpha_1)-\Gamma.
\end{aligned}
$$

By the theory of M-matrices (see, e.g., Mao and Yuan (2006) [Theorem 2.10]), we see easily that both are nonsingular M-matrices.

Moreover, when κ is sufficiently large, $\theta_i \approx 1/(2\kappa\alpha_1)$ and $\bar{\theta}_i \approx 1/(\kappa\alpha_1(q_1 + 1))$ for all $i \in S$. It then easy to see (2.6) is satisfied.

In other words, for a sufficiently large number κ, the control function $u(x, i, t) = Ax^T$ meets Rule 6 as long as $\lambda_{\max}(A) \le -(\kappa + 1)\alpha_1$.

Rule 7

Find eight positive constants ρ_j ($1 \le j \le 8$) with $\rho_4 > \rho_5$ and $\rho_6 \in (0,1)$, and a function $W \in C(R^n; R_+)$, such that

$$\begin{aligned} &\mathcal{L}_2 U(x,y,i,t) + \rho_1 \big(2\theta_i |x| + (q_1+1)\bar{\theta}_i |x|^{q_1}\big)^2 \\ &\quad + \rho_2 |f(x,y,i,t)|^2 + \rho_3 |g(x,y,i,t)|^2 \\ &\qquad \le -\rho_4 |x|^2 + \rho_5 |y|^2 - W(x) + \rho_6 W(y), \end{aligned} \tag{2.9}$$

and

$$\rho_7 |x|^{p+q_1-1} \le W(x) \le \rho_8 (1 + |x|^{p+q_1-1}) \tag{2.10}$$

for all $(x,y,i,t) \in R^n \times R^n \times S \times R_+$.

Rule 8

The time lag τ satisfies

$$\tau < \frac{\sqrt{(\rho_4 - \rho_5)\rho_1}}{2\beta^2}, \quad \tau \le \frac{\sqrt{\rho_1\rho_2}}{\sqrt{2}\beta} \wedge \frac{\rho_1\rho_3}{\beta^2} \wedge \frac{1}{4\beta}. \tag{2.11}$$

Outline

Theorem 9

Under Assumptions 1 and 2, we can design a control function u to satisfy Rules 4 and 6 and then find eight positive constants ρ_j $(1 \le j \le 8)$ *and a function* $W \in C(R^n; R_+)$ *to satisfy Rule 7. If we further make sure* τ *to be sufficiently small for Rule 8 to hold, then the solution of the controlled SDDE (1.3) with the initial data (1.4) has the property that*

$$\int_0^\infty \mathbb{E}|x(t)|^{\bar{q}} dt < \infty, \quad \forall \bar{q} \in [2, p + q_1 - 1]. \tag{2.12}$$

That is, the controlled system (1.3) is H_∞*-stable in* $L^{\bar{q}}$ *for any* $\bar{q} \in [2, p + q_1 - 1]$.

Theorem 10

Under the same conditions of Theorem 9, the solution of the controlled hybrid SDDE (1.3) with the initial data (1.4) has the property that

$$\lim_{t\to\infty} \mathbb{E}|x(t)|^{\bar{q}} = 0, \quad \forall \bar{q} \in [2, q) \tag{2.13}$$

That is, the controlled system (1.3) is asymptotically stable in $L^{\bar{q}}$ for any $\bar{q} \in [2, q)$.

Theorem 11

Under Assumptions 1 and 2, we can design a control function u to satisfy Rules 4 and 6 and then find eight positive constants ρ_j $(1 \le j \le 8)$ and a function $W \in C(R^n; R_+)$ to satisfy Rule 7. If we further make sure

$$\tau < \frac{\sqrt{(\rho_4 - \rho_5)\rho_1}}{2\beta^2} \quad \text{and} \quad \tau \le \frac{\sqrt{\rho_1 \rho_2}}{\sqrt{2}\beta} \wedge \frac{\rho_1 \rho_3}{\beta^2} \wedge \frac{1}{4\sqrt{2}\beta}, \tag{2.14}$$

then the solution of the controlled SDDE (1.3) with the initial data (1.4) has the property that for any initial value $x(0) = x_0 \in R^n$,

$$\limsup_{t\to\infty} \frac{1}{t} \log(\mathbb{E}|x(t)|^{\bar{q}}) < 0, \quad \forall \bar{q} \in [2, q). \tag{2.15}$$

That is, the controlled SDDE (1.3) is exponentially stable in $L^{\bar{q}}$.

Theorem 12

Let all the conditions of Theorem 11 hold. Then the solution of the controlled system (1.3) with the initial data (1.4) has the property that

$$\limsup_{t\to\infty} \frac{1}{t} \log(|x(t)|) < 0 \quad a.s. \tag{2.16}$$

That is, the controlled SDDE (1.3) is almost surely exponentially stable.

Comments

The results established in this paper are all independent of δ, which is the time lag of the given SDDE, but very much dependent on τ, which is the time lag between the time when the state is observed and the time when the feedback control reaches the system.

Rule 6 describes a way how to find positive numbers θ_i and $\bar{\theta}_i$ ($i \in S$) and then further to find positive numbers ρ_j ($1 \le j \le 8$) in Rule 7. On the other hands, if one can find all these positive numbers for Rule 7 to be satisfied, then all of our results hold without Rule 6.

The control function u used in this paper is allowed to depend on mode i, namely we use $u(x, i, t)$. This enables us to make use of different system structure in different mode to design the control function more wisely. It is possible to use a simpler control function which depends on the state x only, namely $u(x)$, for example, $u(x) = Ax$. Of course, this is applicable only in the situation where the state is observable and the feedback control can be input in every mode.

In some situation where the state of the underlying system is not observable in some modes, we have to design the feedback control function only on those modes which state is observable and put no control on the other modes. The examples discussed in the next section illustrate these situations fully.

Outline

We have discussed the stabilisation of highly nonlinear hybrid SDDEs by the delay feedback controls.

We pointed out that there is little known on this stabilisation problem when the feedback control is based on the past state although the feedback control based on the current state has been well studied.

We also pointed out that the problem becomes even harder when the coefficients of the underlying hybrid SDDE do not satisfy the linear growth condition (namely, the coefficients are highly nonlinear).

We consider a class of hybrid SDDEs which are not stable but their solutions are bounded in qth moment.

We then propose four rules for the control functions such that the controlled SDDEs become stable.

These rules, to a very much degree, also describe a way how to design the control functions.

The stability discussed include the H_∞-stable in $L^{\bar{q}}$, asymptotic stability in $\bar{q}$th moment, qth moment exponential stability and almost surely exponential stability.

The key technique used in this paper is the method of Lyapunov functionals.